# Mechanism of the Si$(d,p)$ Reactions Below 3 MeV*

B. H. WILDENTHAL,† R. W. KRONE, AND F. W. PROSSER, JR.
*University of Kansas, Lawrence, Kansas*

(Received 5 February 1964; revised manuscript received 30 April 1964)

The $Si^{28}(d,p)Si^{29}$ and $Si^{30}(d,p)Si^{31}$ reactions were studied in the deuteron energy range from 2 to 3 MeV. Excitation functions for the reactions leading to various final states were obtained at 30 and 135° in the laboratory system. The differential cross sections exhibited fluctuations from 40 to 200 keV wide which were uncorrelated between the two angles of measurement and between the various final states of a nucleus. Angular distributions and the resulting total cross sections were obtained at fourteen deuteron energies. These data fluctuated significantly over short energy ranges. Plane-wave Born-approximation analysis did not give consistently correct values of $l_n$ for $l_n>0$. Distorted-wave Born-approximation predictions were fitted to the experimental distributions with reasonable success for the $l_n=0$ and 1 transitions and with less conclusive results for the $l_n=2$ transitions. The transition to the sixth excited state (3.54 MeV) of $Si^{31}$ was assigned the value $l_n=1$. These theoretical predictions were used as a basis of estimating the relative importance of compound nucleus and direct-interaction reaction mechanisms in this energy range. The possibility of explaining the observed fluctuations in terms of the Ericson fluctuations of the compound-nucleus reaction component was explored.

## INTRODUCTION

THE details of the mechanism of the $(d,p)$ reactions involving medium light nuclei are not well understood for bombarding energies below the Coulomb barrier. Measurements of differential cross sections as functions of the deuteron energy show significant fluctuations which are inexplicable in terms of an ordinary direct reaction interpretation.[1–3] These fluctuations have been attributed to resonances in the compound system and to interference between the direct and compound-system modes of reaction.[1–5] Estimates of the relative importance of the different mechanisms have been made, based on several lines of reasoning.[1,2,6] Unfortunately, the theoretical tools for analyzing experimental situations in which there are a number of compound states competing strongly with the direct mode are not well developed.[7] The angular distributions of the protons can provide information both about the particular direct reactions involved and the competition with compound-system formation. Naturally the presence of compound-system effects hinders definitive conclusions about the details of the direct reaction mechanism.

Although some angular distributions of protons from low-energy deuteron reactions have been fairly well described by the Butler or plane-wave Born approximation (PWBA) form of stripping theory,[8–11] it is generally agreed that the distorted-wave Born approximation[9] (DWBA) should always be more accurate. This is true especially with medium light and heavier target nuclei for which the Coulomb and nuclear distortions of the incident and emergent wavepackets are most difficult to ignore. Angular distributions completely inexplicable in terms of the plane-wave theory have been well described by distorted-wave analysis.[12–13] A broad survey of the effectiveness of the distorted-wave Born approximation in describing $(d,p)$ reactions indicates that, while for heavier $(A>45)$ nuclei the theoretical agreement with experiment is quite good, various inconsistencies become evident when the theory is applied to lighter nuclei.[14,15] These inconsistencies are in all probability related to the previously mentioned fluctuations in the differential cross section.

The $Si^{28}(d,p)Si^{29}$ and $Si^{30}(d,p)Si^{31}$ reactions were studied in the deuteron energy range from 2 to 3 MeV to obtain information about the reaction mechanisms involved and the limits of usefulness of the distorted-wave Born-approximation analysis in this energy and mass range. Excitation functions for the reactions to the various final states were measured to determine the magnitude of the cross-section fluctuations and the energies at which they occurred. Angular distributions were measured for various values of the incident energy to ascertain the relations between the characteristics of

* This work has been supported in part by the U. S. Atomic Energy Commission.

† Present address: Rice University, Houston, Texas.

[1] L. L. Lee, Jr., and J. P. Schiffer, Phys. Rev. **107**, 1340 (1957).

[2] J. A. Kuehner, E. Almqvist, and D. A. Bromley, Nucl. Phys. **21**, 555 (1960).

[3] Iu. A. Nemilov and V. F. Litvin, Zh. Eksperim. i Teor. Fiz. **31**, 719 (1956) [English transl.: Soviet Phys.—JETP **4**, 606 (1957)].

[4] J. B. Teplov and B. A. Iur'ev, Zh. Eksperim. i Teor. Fiz. **34**, 334 (1958)) [English transl.: Soviet Phys.—JETP **7**, 233 (1958)].

[5] J. P. Schiffer and L. L. Lee, Jr., Phys. Rev. **115**, 1705 (1959).

[6] L. Colli and F. Tonolini, Phys. Letters **3**, 149 (1962).

[7] N. Austern, in *Selected Topics in Nuclear Theory*, edited by F. Janouch (International Atomic Energy Agency, Vienna, 1963).

[8] S. T. Butler, Proc. Roc. Soc. (London) **A208**, 559 (1951).

[9] W. Tobocman, *Theory of Direct Nuclear Reactions* (Oxford University Press, London, 1961).

[10] D. J. Pullen, D. H. Wilkenson, and A. B. Whitehead, in *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961*, edited by J. B. Birks (Academic Press Inc., New York, 1961), p. 565.

[11] V. M. Rout, W. M. Jones, and D. G. Waters, Nucl. Phys. **45**, 369 (1963).

[12] W. Tobocman, Phys. Rev. **115**, 98 (1959).

[13] W. Tobocman and W. R. Gibbs, Phys. Rev. **126**, 1076 (1962).

[14] W. R. Smith and E. V. Ivash, Phys. Rev. **128**, 1175 (1962).

[15] W. R. Smith and E. V. Ivash, Phys. Rev. **131**, 304 (1963).

the distributions and the structure of the excitation functions. The large number of angular distribution measurements increased the probability of observing cases in which the distribution from the direct component of the reaction was relatively uncontaminated by compound nucleus effects; also, they provided the total cross sections which were used to study the applicability of the statistical theory predictions[16] to this type of data. The predictions of the DWBA were instrumental in the analysis and allowed a more accurate determination of the relative strengths of the direct interaction and compound nucleus contributions to the total cross section than has been available with PWBA predictions.

## EXPERIMENTAL PROCEDURE

The beam of charged particles used in this experiment was obtained from a High-Voltage Engineering Corporation 3-MeV Van de Graaff accelerator. The beam energy was determined by measuring the strength of the deflecting magnetic field with a standard NMR device. The system was calibrated with the $Li^7(p,n)Be^7$ reaction, using $E_p=1880.6\pm0.3$ keV as the value of threshold energy.[17] The beam was collimated to an area of approximately 0.04 cm$^2$ and limited to a maximum current of 1 $\mu$A. Target backings were constructed of gold leaf (190 $\mu$g/cm$^2$) mounted over 1.3-cm apertures in tantalum blanks. Natural silicon or silicon enriched[18] with $Si^{30}$ was evaporated onto the backings; experience showed that the targets prepared in this manner did not deteriorate appreciably with the beam currents used. Carbon buildup was minimized with cold traps which isolated the chamber from all pumping systems, but was still so severe as to make impractical detailed measurements of silicon groups occurring in the region of the $C^{12}(d,p)C^{13}(0)$ proton group.

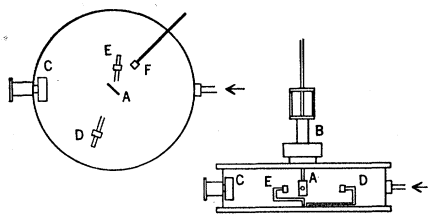


FIG. 1. Line drawing of the reaction chamber and associated equipment. A denotes the target, B the target-post–air-lock assembly, C the Faraday cup, D and E the rotating counters, and F the fixed counter.

The experiment was performed in a large chamber designed for charged particle reaction studies.[19] The body of the chamber is a cylinder 15.3 cm high and 61 cm inside diameter as shown in Fig. 1. Targets are mounted on a retractable, rotating post that enters the chamber through the air lock arrangement on the top plate. Two silicon surface-barrier particle detectors[20] were mounted to rotate about the target at radial distances of 6.2 and 16.2 cm. The angular position of these detectors is controlled and monitored remotely to within $\pm\frac{3}{4}°$, These detectors were employed to measure the angular distributions, data being taken in 10° steps $\theta$ from +90 to +170° and from −10 to −90°. For use primarily in measuring the excitation functions, an

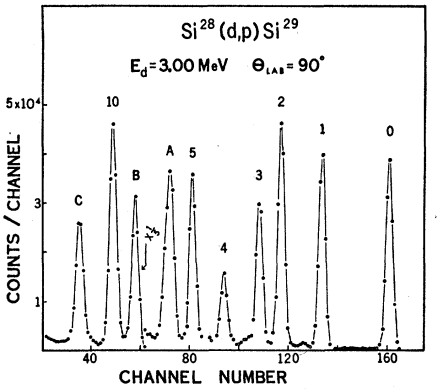


FIG. 2. Typical spectrum of the proton groups from the deuteron bombardment of a natural silicon target, recorded on the multichannel pulse-height analyzer. The numbers 0, 1, 2, etc., denote the proton groups corresponding to the ground state, first excited state, and so forth of $Si^{29}$. "A" denotes the proton group from the $C^{12}(d,p)C^{13}(0)$ reaction. The presence of the $Si^{29}(6)$ group contributes to the broadening of this peak. B and C mark the proton groups from the $O^{16}(d,p)O^{17}$ reactions leading to the ground and first excited states of $O^{17}$.

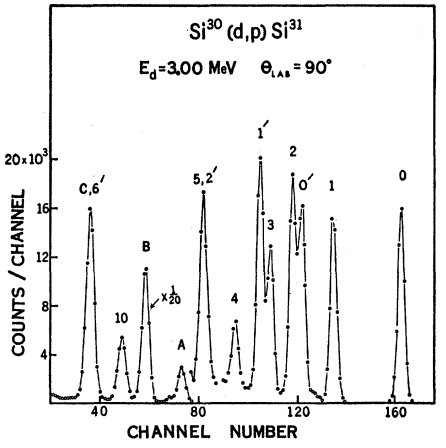


FIG. 3. Typical spectrum of the proton groups from deuteron bombardment of a $Si^{30}$-enriched target. Protons groups from the $Si^{30}(d,p)Si^{31}$ reactions leading to the ground state, first, second, and sixth excited states of $Si^{31}$ are denoted by 0′, 1′, 2′, and 6′.

[16] T. Ericson, Advan. Phys. 9, 425 (1960).
[17] F. C. Young and J. B. Marion, Nucl. Phys. 41, 561 (1963).
[18] Silicon enriched to 68% $Si^{30}$ was obtained from Oak Ridge National Laboratory.
[19] B. H. Wildenthal, thesis, University of Kansas, 1964 (unpublished).
[20] The rotating detectors were ORTEC model SBFJ, 0.25 cm$^2$ in active area with a 500-$\mu$ depletion depth.

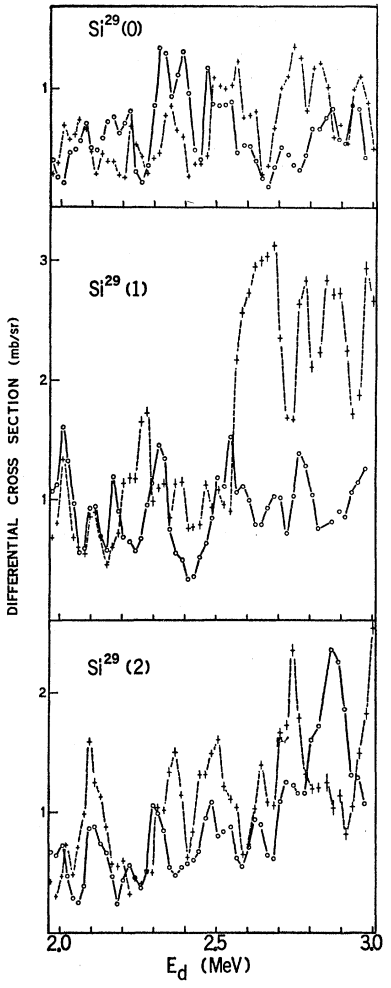


FIG. 4. Excitation functions of the $Si^{28}(d,p)Si^{29}$ reactions leading to the ground, first excited, and second excited states of $Si^{29}$. The data taken at 30° in the laboratory system is denoted by the cross marks that taken at 135° is indicated by the open circles. The connecting lines are explicitly given only to aid the eye. The vertical extension of the points is a measure of the statistical errors only.

additional detector[21] was mounted at the 135° port hole on a sliding arm that allowed the detector to be positioned anywhere from 1 to 25 cm from the target. Gold or tantalum foils of thickness sufficient to stop the elastically scattered deuterons were mounted in front of the detectors. The associated electronics consisted of Hamner N-357 preamplifiers and a TMC-404 400-channel pulse-height analyzer. Normally, several spectra were simultaneously accumulated in 100- or 200-channel sections of the analyzer.

The beam was collected in a Faraday cup. An intermediate electrode was biased to −300 V to provide electrostatic suppression of backscattered and knock-on electrons. In addition, the target post was biased to a positive potential of 300 V to suppress electrons discharged from the target by the beam. The collected charge of the beam was measured with an Elcor A309B current indicator and integrator. The thickness of the target used in measuring the $Si^{28}$ absolute differential cross sections was measured by weighing the amount of silicon deposited on thin aluminum strips which were placed in the evaporation chamber so as to receive a coating of silicon equal to that recieved by the target. The weighing procedure gave results reproducible to $\pm 1$ $\mu g/cm^2$. The target used in the absolute cross section determination was judged in this manner to have a thickness of $13\pm 2$ $\mu g/cm^2$ and the mass was assumed to be pure natural silicon. The error assigned includes an estimate of the uncertainity in the uniformity of the deposited layer. The error in the scale of the cross sections, a root-square value derived from uncertainties in the target thickness, charge collection, solid angle measurements, and counting statistics, was estimated at 18%. The extent to which oxygen, visible in the experimental spectra, contributed to the weight of the sample, is an unknown quantity, introducing a maximum possible correction of less than 10%. The thickest targets employed in the experiment were less than 10 keV thick to 2-MeV deuterons. The excitation functions were measured with targets of approximately 5-keV equivalent thickness.

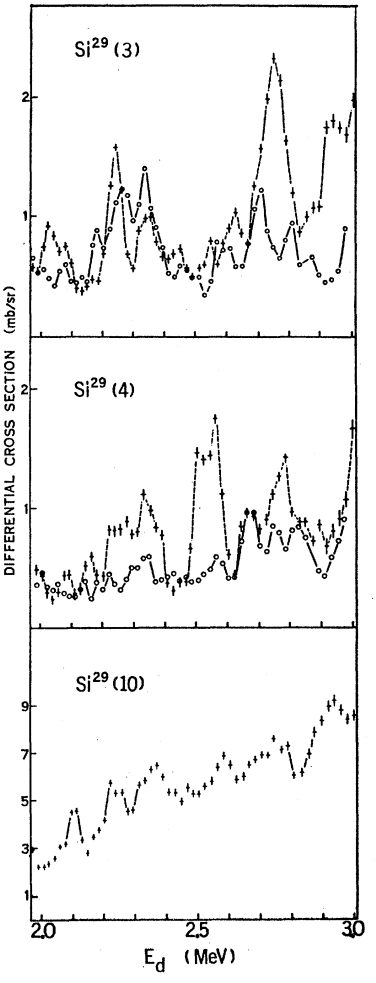


FIG. 5. Excitation functions of the $Si^{28}(d,p)Si^{29}$ reactions leading to the third, forth, and tenth excited states of $Si^{29}$. Data obtained at 30° in the laboratory system is indicated by cross marks. Open circles indicate the data taken at 135°. The lines are intended only to aid to the eye in following the data.

[21] The fixed detector was an RCA model C-4 ,0.20 $cm^2$ in active area with a 450-$\mu$ depletion depth.

## EXPERIMENTAL RESULTS

Typical proton spectra of the $Si^{28}(d,p)Si^{29}$ and $Si^{30}(d,p)Si^{31}$ reactions are shown in Figs. 2 and 3. The energy calibration of the 400-channel analyzer was based on the proton groups from the carbon and oxygen contaminants, and identification of the various Si groups was made with the aid of previous energy level investigations.[22,23] The intensities of the transitions to the various levels of the residual nuclei were measured by summing the total number of counts in the respective peaks. The $Si^{30}(d,p)Si^{31}$ results were obtained by subtracting normalized spectra of natural silicon from the spectra obtained from the isotopically enriched targets. Where feasible, a similar technique was employed in disentangling silicon groups from the contaminant groups.

The differential cross sections of the $Si^{28}(d,p)Si^{29}$ and $Si^{30}(d,p)Si^{31}$ reactions leading to various levels of the residual nuclei were measured simultaneously at 30 and 135°. The excitation functions in the deuteron energy range from 2 to 3 MeV are presented in Figs. 4–6. The total cross sections, obtained from the integration of the angular distribution measurements at fourteen values of incident deuteron energy, are presented in Fig. 7. The specific reactions investigated are those leading to the ground state, first (1.277 MeV), second (2.027 MeV), third (2.424 MeV), fourth (3.067 MeV), and tenth (4.931 MeV) excited states of $Si^{29}$ and to the ground state and first excited state (0.76 MeV) of $Si^{31}$. In the remainder of the discussion, the various states of the reactions forming them will be referred to with the convention in which, e.g., $Si^{29}(3)$ denotes the reaction forming the third excited state of $Si^{29}$.

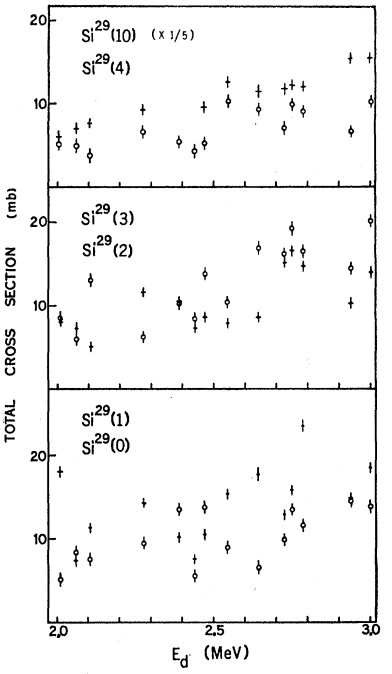


FIG. 7. Total cross sections of the $Si^{28}(d,p)Si^{29}$ reactions at deuteron energies of 2.010, 2.062, 2.106, 2.276, 2.390, 2.440, 2.472, 2.545, 2.643, 2.726, 2.750, 2.785, 2.938, and 3.001 MeV. Values for the $Si^{29}(0)$, $Si^{29}(2)$ and $Si^{29}(4)$ reactions are given by the open circles. The cross marks indicate the data from the $Si^{29}(1)$, $Si^{29}(3)$, and $Si^{29}(10)$ reactions.

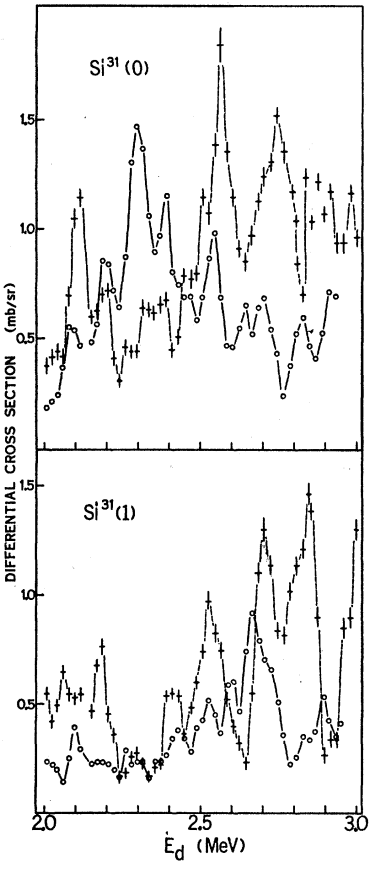


FIG. 6. Excitation functions of the $Si^{30}(d,p)Si^{31}$ reactions leading to the ground and first excited states of $Si^{31}$. Data taken at 30° in the laboratory system is indicated by cross marks. The open circles indicated the data obtained at 135°. The lines are intended only as an aid to the eye.

The data are characterized by large and random fluctuations in cross section. The maxima and minima in the differential cross section of the reaction to a given level measured at 30° are generally not correlated with the structure measured at 135°. Furthermore, there are no apparent correlations in differential or total cross sections of reactions leading to different levels of the final nuclei. This differs in a fundamental way from results obtained with light nuclei[24–26] where isolated individual resonances in the compound state may be distinguished by correlated changes in the various decay channels. The fairly straightforward analysis[27,28] which separates the direct and compound-nucleus contributions in such a case is therefore not available for the present experiments. Also, these results differ from

[22] D. M. Van Patter and W. W. Buechner, Phys. Rev. **87**, 51 (1952).
[23] C. P. Browne and J. T. Radzyminski, Nucl. Phys. **19**, 164 (1960).
[24] J. P. F. Sellschop, Phys. Rev. **119**, 251 (1960).
[25] E. Kashy, R. R. Perry, and J. R. Risser, Phys. Rev. **117**, 1289 (1960).
[26] J. M. F. Jeronymo, G. S. Mani, F. Picard, and A. Sadegbi, Nucl. Phys. **43**, 417 (1963).
[27] S. Yoshida, in *Proceedings of the International Conference on Nuclear Structure, Kingston, Ontario*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, 1960), p. 336.
[28] T. W. Bonner, J. T. Eisinger, A. A. Kraus, Jr., and J. B. Marion, Phys. Rev. **101**, 209 (1956).

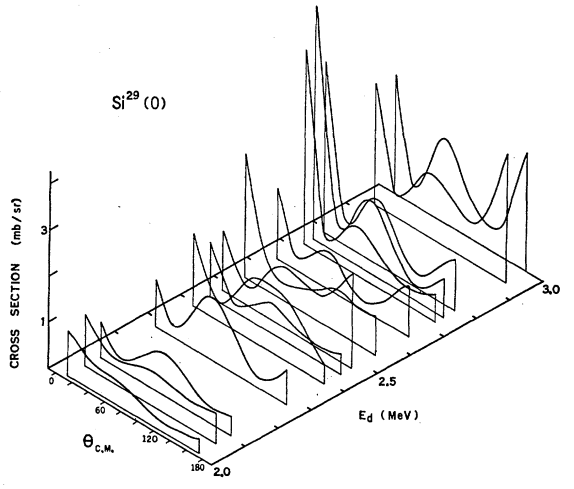


FIG. 8. Angular distribution of protons from $Si^{28}(d,p)Si^{29}(0)$ at the deuteron energies 2.010, 2.062, 2.106, 2.276, 2,390. 2.440, 2.472, 2.545, 2.643, 2.726, 2.750, 2.785, 2.938, and 3.001 MeV. The data was obtained in 10° steps from 10 to 170° and is presented in the center-of-mass system.

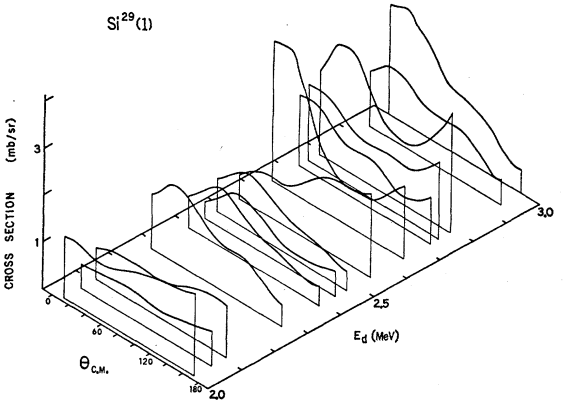


FIG. 9. Angular distributions of protons from $Si^{28}(d,p)Si^{29}(1)$, obtained in the manner of the $Si^{29}(0)$ distributions.

measurements of the $Si^{28}(d,p)Si^{29}$ reaction from 6- to 10-MeV deuteron energy[2] in that the present data show considerable fluctuation in the total cross sections. The excitation functions are similar to those published by Lee and Schiffer[1] for the ground state of $Ca^{41}$, and indicate that measurements taken with coarser energy steps miss most of the structure of the fluctuations.[3]

The characteristics of the angular distributions of the protons from the various reactions, as functions of the incident energy, are shown in Figs. 8–15. These angular distributions have all been investigated at higher deuteron energies and the values of the orbital angular momentum of the captured neutron $l_n$ forming the residual levels have been extracted by PWBA analysis.[29–31] $Si^{29}(0)$ and $Si^{31}(1)$ are assigned $l_n=0$, $Si^{29}(10)$ is assigned $l_n=1$, and $Si^{29}(1)$, $Si^{29}(2)$, $Si^{29}(4)$ and $Si^{31}(0)$ are assigned $l_n=2$. The angular distribution of $Si^{29}(3)$ is isotropic in the higher energy measurements and the reaction has been assumed not to proceed through the direct reaction mode. In the present measurements, the angles of maximum cross section in a distribution vary irregularly with deuteron energy. The 0° peak of the $l_n=0$ transitions is the most stable feature found in the various distributions, but even in this instance, the differential cross section at 10° fluctuates by as much as a factor of 3. In general, intensity fluctuations at the PWBA stripping peak angles are as intense as those at backward angles. Hence, as has been noted,[7] attempts to divide the cross section into a slowly varying direct interaction contribution, found mainly under the stripping peak and a fluctuating compound-nucleus contribution, symmetric about 90° and most noticeable in the backward direction, appear to be complicated.

## DISCUSSION

### General Procedure

The analysis of the data involves the relationship between the excitation function of a given final-state

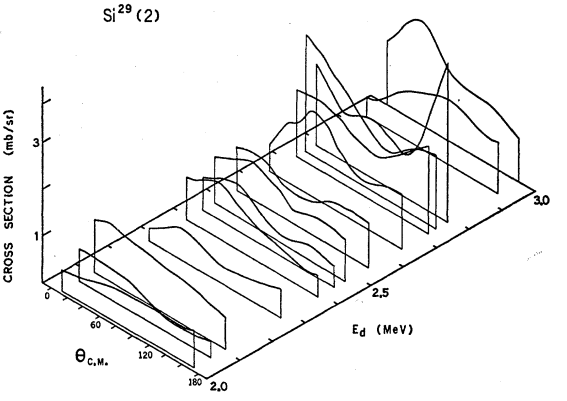


FIG. 10. Angular distributions of protons from $Si^{28}(d,p)Si^{29}(2)$, obtains in the manner of the $Si^{29}(0)$ distributions.

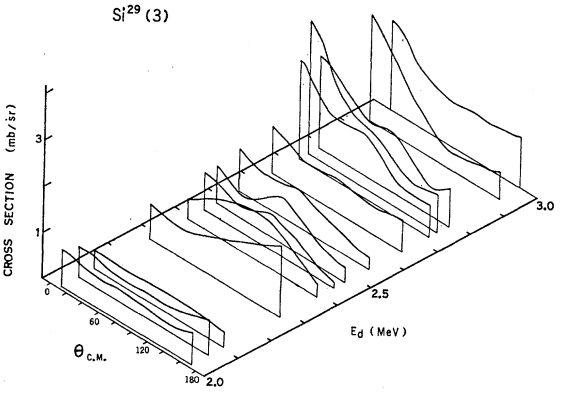


FIG. 11. Angular distributions of protons from $Si^{28}(d,p)Si^{29}(3)$, obtained in the manner of the $Si^{29}(0)$ distributions.

[29] J. R. Holt and T. N. Marsham, Proc. Phys. Soc. (London) **A66**, 467 (1953).
[30] V. G. Sukharevskii, Zh. Eksperim. i Teor, Fiz. **36**, 52 (1959) [English transl.: Soviet Phys.—JETP **9**, 37 (1959)].
[31] A. G. Blair and K. S. Quisenberry, Phys. Rev. **122**, 869 (1961).

reaction and the angular distributions of this reaction at various deuteron energies. The basic problem is the division of the observed total reaction probability into a direct interaction (DI) component, a compound nucleus (CN) component and, possibly, a direct-compound interference term. There is no straightforward theoretical framework available with which to treat the complex data of this experiment. A heuristic approach to the analysis has been attempted, with the consistency of the conclusions being taken as a test of the soundness of the premises.

Theoretical angular distributions of the protons from the various reactions were calculated with the distorted-wave Born approximation. Values of the optical-model parameters were chosen to be consistent, as far as possible, with the choices of related analyses, and were not varied appreciably for the different values of $l_n$, $E_d$, and $Q$. These theoretical predictions were compared with the various experimental distributions and an attempt was made to discern a correlation between good theoretical-experimental agreement and the structure of the related excitation function. The assumption was that agreement of the experimental distribution with the DWBA theory indicated a relatively small contribution from the compound-nucleus component of the total reaction cross section.

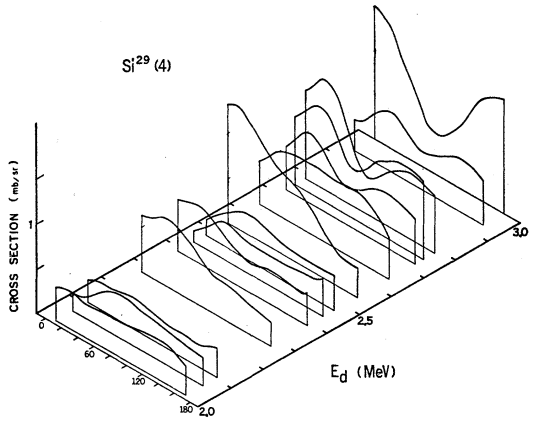


FIG. 12. Angular distributions of protons from $Si^{28}(d,p)Si^{29}(4)$, obtained in the manner of the $Si^{29}(0)$ reactions.

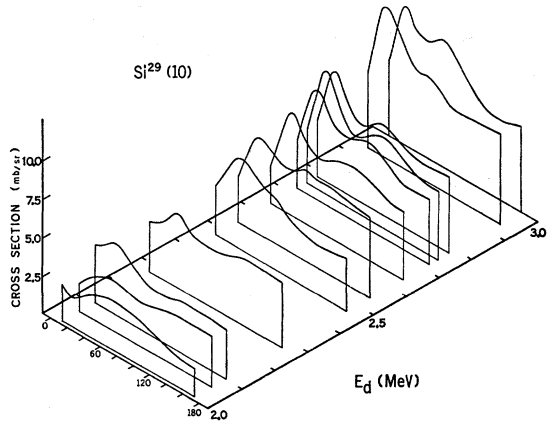


FIG. 13. Angular distributions of protons from $Si^{28}(d,p)Si^{29}(10)$, obtained in the manner of the $Si^{29}(0)$ reaction.

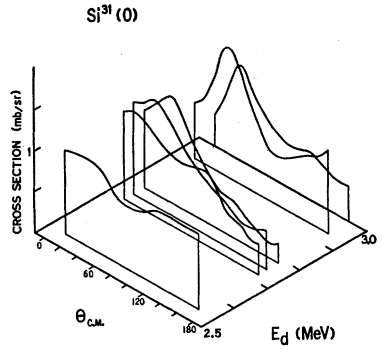


FIG. 14. Angular distributions of protons from $Si^{30}(d,p)Si^{31}(0)$. Normalized $Si^{28}$ spectra were substracted from the $Si^{30}$ enriched data to obtain the distributions. The deuteron energies at which data are shown are 2.545, 2.726, 2.750, 2.785, 2.938, and 3.001 MeV.

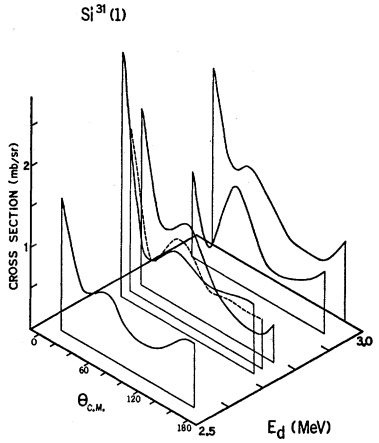


FIG. 15. Angular distributions of protons from $Si^{30}(d,p)Si^{31}(1)$. The data was obtained in the same manner as the $Si^{31}(0)$ data.

## DWBA Calculations

The DWBA calculations were performed with the computer code developed by Smith.[32] This was essentially the program used in the extensive surveys of Smith and Ivash,[14,15] and the analysis was performed here in accordance with their procedures. The form factors of the Woods-Saxon wells were generally held constant to the values $R_{0p}=1.25$ F, $R_{0d}=1.4$ F, $a_p=0.5$ F, and $a_d=0.7$ F. Smith and Ivash have discussed the effect of various types of parameter variations upon the theoretical distributions. The results obtained in the present work were in general conformity with their discussion.

The general rules followed in varying the parameters were in part dictated by physical reasonableness and in part by previous empirical success and computational efficiency. The $VR^2$ degeneracy was taken as the

[32] W. R. Smith (private communication).

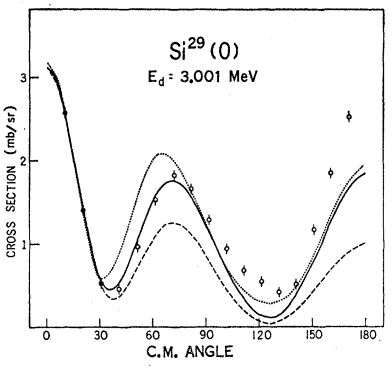


FIG. 16. The data points are from the $Si^{29}(0)$ reaction at $E_d=3.001$ MeV. The solid line is the DWBA prediction for this reaction based upon the parameter set C. The predictions based upon set A are represented by the dotted line and the dashed line represents the predictions based upon set D. For simplicity and consistancy, the theoretical curves here and in the succeeding figures are normalized to the main experimental peak.

justification for holding constant the spatial dimensions of the wells. The value of $V_p$ was, in general, varied only slightly about the value of 52 MeV, a value consistent with previous work,[15] since variation of $V_p$ seemed to produce results similar to those obtained by variation of $V_d$. The ratio of the depths of the imaginary potentials $W_d$ and $W_p$ was held at one of the values $W_d/W_p=2$ or $\frac{4}{3}$. The main parameter search thus centered upon finding combinations of $V_d$ and $W_d$ that produced the best fits to the experimental data. Since the main characteristics of the distribution can be altered in the same general manner by different changes in the different parameters, the final set of parameters, even after a thorough search, cannot be considered completely definitive. For example, the first minimum may be shifted to a larger angle by decreasing $R_p$, $R_d$, $V_p$, or $V_d$, or by increasing $W_d$ and $W_p$. Therefore, the value of each parameter must be viewed in its relation to all the others. Parameter combinations which yielded fair to good approximations to the data of this experiment are listed in Table I.

The $l_n=0$ transitions to $Si^{29}(0)$ were used as the basis for selecting the best optical-model parameters. Previous investigators[13–15,33] have found better agreement between DWBA predictions and experimental results for low values of $l_n$ and particularly $l_n=0$; also, the structure of the $l_n=0$ angular distributions exhibit a much more definitive structure at the energies of the present experiment than do distributions involving higher angular momentum capture. The positions of the first minimum and the second maximum and the relative heights of the three maxima were considered the most important characteristics in the evaluation of the agreement between theory and experiment. Good qualitative agreement and consistency were emphasized, rather than strict quantitative fits. The parameter combinations used were, in general those suggested by the analysis of this reaction at $E_d=4$ MeV [4] by Smith and Ivash.[15] Their chosen combination of parameters is set A of Table I. Their results for light nuclei indicate a general tendency of $V_d$ to decrease with $E_d$.

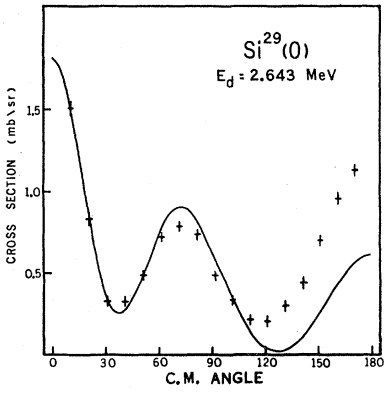


FIG. 17. The data points are from the $Si^{29}(0)$ reaction at $E_d=2.643$ MeV. The solid line represents the DWBA predictions for this reaction at $E_d=2.750$ MeV, based upon parameter set D.

TABLE I. Combinations of optical-model parameters employed in the computations presented in the various figures. The spatial dimensions of the Woods-Saxon wells were held constant at the values $R_{0d}=1.4$F, $R_{0p}=1.25$ F, $a_d=0.7$ F, $a_p=0.5$ F.

| Set | $V_d$ (MeV) | $V_p$ (MeV) | $W_d$ (MeV) | $W_p$ (MeV) |
|---|---|---|---|---|
| A | 68 | 53 | 8 | 6 |
| B | 72 | 52 | 8 | 6 |
| C | 63 | 53 | 8 | 6 |
| D | 60 | 52 | 12 | 6 |
| E | 60 | 52 | 18 | 12 |

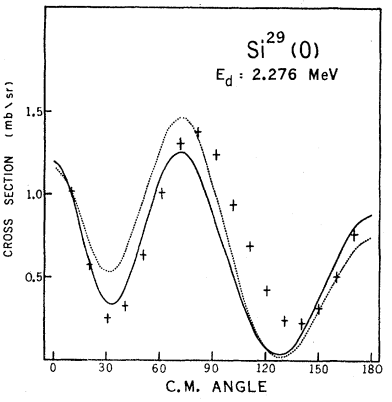


FIG. 18. The data points are from the $Si^{29}(0)$ reaction at $E_d=2.276$ MeV. The solid and dotted lines represent, respcetively, the DWBA predictions for this reaction at $E_d=2.469$ and 2.062 MeV, based upon parameeter set C.

## DWBA—Experimental Comparisons

*$l_n=0$ transitions.* The comparison of the experimental angular distributions of protons from the reaction $Si^{28}(d,p)Si^{29}(0)$ with the distributions predicted by the DWBA theory using various parameter combinations indicated that consistent acceptable agreement between theory and experiment could be obtained only for those experimental distributions measured at deuteron ener-

[33] B. Buck and P. W. Hodgson, Nucl. Phys. **29**, 496 (1962).

gies for which the differential and total cross sections of this reaction showed minima. Examples of the better theoretical fits to these experimental distributions are shown in Figs. 16–18. Correspondingly, experimental angular distributions measured at energies for which the excitation functions and total cross sections of the $Si^{29}(0)$ reaction reached maxima, e.g., $E_d=2.390$ and 2.472 MeV, could not be well fitted by DWBA predictions based upon reasonable parameter combinations.

The inference to be drawn from these results seems to be that the observed reaction probability is composed of a fairly incoherent mixture of a slowly varying DI component and a fluctuating CN component in which occur the rapid changes in intensity and distribution which characterize the data. This composition of the experimental cross sections was assumed in the remainder of the DWBA analysis of the data. Only angular distributions of a reaction which were measured at excitation function minima were expected to be well approximated by the theoretical predictions. Although

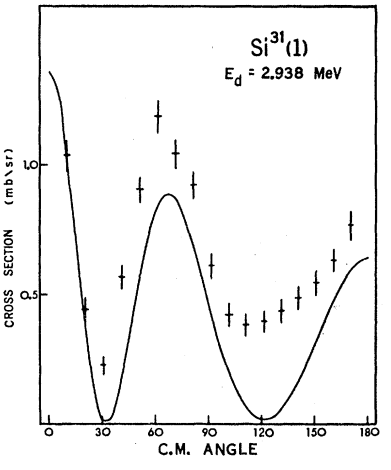


FIG. 19. The data points are from the $Si^{30}(d,p)Si^{31}$ reaction at $E_d=2.938$ MeV. The solid line represents the DWBA prediction for this reaction at $E_d=3.001$ MeV, based upon parameter set C.

this approach did not yield uniformly successful results, no other approach to the problem was as unambiguous and consistent. Several experimental and theoretical points which are pertinent to the justifiability of this assumption are developed in following sections.

The data of the $Si^{30}(d,p)Si^{31}$ reaction are in agreement with the foregoing assumptions. The angular distribution measured at $E_d=2.938$ MeV, an energy for which the excitation functions of this reaction show extreme minima, is well fitted by DWBA predictions which employ parameters consistent with those chosen in the $Si^{29}(0)$ analysis (see Fig. 19). Angular distributions of this reaction measured at excitation function maxima were unlike both theoretical predictions and the $Si^{28}(d,p)Si^{29}(0)$ experimental results.

*$l_n=1$ transitions.* The relatively large cross section of the $Si^{28}(d,p)Si^{29}(10)$ reaction, also apparent in the energy range of the present experiment, led previous

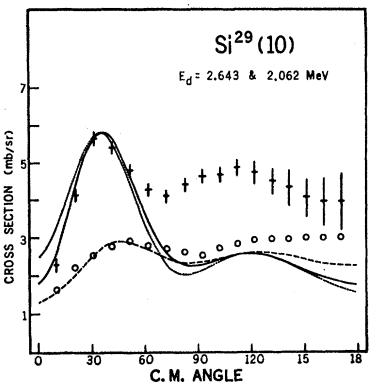


FIG. 20. The data from the $Si^{29}(10)$ reaction at $E_d=2.643$ MeV are represented by the cross marks. The dotted and solid lines represent the DWBA predictions for this reaction at $E_d=2.750$ MeV, based upon parameter sets C and D, respectively. The open circles represent the data from the $Si^{29}(10)$ reaction at $E_d=2.062$ MeV and the dashed line represents the DWBA prediction for this reaction at $E_d=2.062$ MeV, based upon parameter set C.

investigators[29,31,34] to assign a rather pure single-particle character to the final state. As such, the angular distribution should be relatively less contaminated by CN effects and hence fairly well approximated by the DWBA theory at all energies. In agreement with this supposition, the excitation function at 30° [the $O^{16}(d,p)O^{17}(0)$ reaction prevented taking data at 135°] shows much smaller relative fluctuations and a steady increase of cross section with $E_d$. Unfortunately, it is difficult to reach an unambiguous conclusion about this reaction because the 9th, 8th, and perhaps the 7th excited states of $Si^{29}$ are not resolved

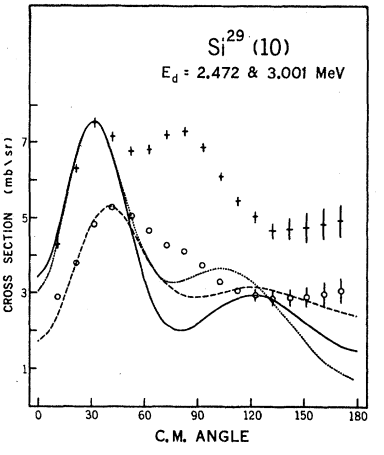


FIG. 21. The data from the $Si^{29}(10)$ reaction at $E_d=3.001$ MeV are indicated by the cross marks. The solid and dotted lines represent the DWBA predictions for this reaction and energy based, respectively, upon parameter sets C and B. The open circles indicate the data of the $Si^{29}(10)$ reaction at $E_d=2.472$ MeV. The dashed line represents the DWBA prediction for this reaction and energy, based upon parameter set D.

[34] M. H. Macfarlane and J. B. French, Rev. Mod. Phys. **32**, 567 (1960).

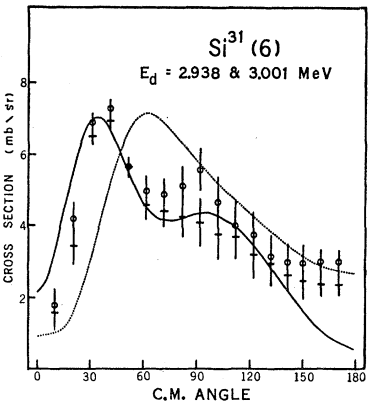


FIG. 22. The data from the $Si^{30}(d,p)Si^{31}(6)$ reaction at $E_d=2.938$ and 3.001 MeV are indicated by the cross marks and circles respectively. The vertical bars indicate the estimated uncertainty in the measurements. These overlap at many angles. The solid line represents the DWBA prediction for this reaction at $E_d=3.001$ MeV, based upon parameter set B and assuming $l_n=1$, $J_f=\frac{3}{2}$. The dotted line represents the prediction based upon the same circumstances expect that $l_n=2$.

from the 10th. High-resolution work indicates that these states have a low relative intensity[22,23]; their contribution to the $Si^{29}(10)$ distribution has been neglected in previous work of this nature.[29,31]

In view of the cross section fluctuations observed in the lower lying states, the possibility that these weaker states influence the shape of the distribution significantly at some points cannot be ignored. Further complications arise from the fact that the oxygen contamination introduces increasingly large probable errors from 120° on backward.

The DWBA predictions for the angular distributions of protons from $Si^{28}(d,p)Si^{29}(10)$ are compared with several experimental distributions in Figs. 20 and 21. Agreement with the position of the main peak is good. The fit is deficient in that the experimental second maximum is usually much larger than in the theoretical distributions. The origin of this discrepancy could either be the presence of the unresolved lower states or the presence of a larger than expected CN contribution. Alternatively, the discrepancy may result from the family of nonvaried parameters chosen for this analysis, or other internal assumptions of the computation such as the shape of the potentials.

The $Si^{30}(d,p)Si^{31}(6)$ reaction was observed to have a large cross relative to the transitions to the lower lying states. The $l_n$ value for this transition has not previously been assigned. Although the proton group from the strong $O^{16}(d,p)O^{17}(1)$ reaction was unresolved from the $Si^{31}(6)$ peak at most angles, special pains were taken to obtain experimental distributions of this reaction. This was found to be feasible at $E_d=2.938$ and 3.001 MeV, since at these energies the oxygen reaction cross section has a minimum. The results are shown in Fig. 22. It appears from inspection that the value for $l_n$ should be 1 or 2. DWBA computations were performed for those spin combinations inferred by these assumptions. A sample of the results, clearly indicating that $l_n=1$, is shown in Fig. 22.

*$l_n=2$ transitions.* Angular distributions of $l_n=2$ reactions measured at excitation functions minima include those of $Si^{29}(1)$ at $E_d=2.938$, 2.750, and 2.440 MeV, $Si^{29}(2)$ at $E_d=2.276$ MeV, and $Si^{29}(4)$ at $E_d=2.938$ and 2.440 MeV. Attempts to fit these distributions using DWBA parameters chosen in the $l_n=0$ and 1 analyses were only marginally successful (Figs. 23, 24, and 25). The most obvious explanation of the difficulty is that even at many excitation function minima, the relative proportion of the CN component is significantly greater than in the $l_n=0$ and $l_n=1$ cases. Previous investigations[2] are in agreement with this supposition. The occasional fair agreement [Fig. 24, $Si^{29}(1)$ at $E_d=2.785$ MeV] between DWBA predictions and angular distributions measured at excitation-function maxima must be regarded as resulting from fortuitous CN distributions. Angular distributions measured at the most pronounced cross-section minima (Fig. 25) required somewhat modified parameter values for the best DWBA fits. The fits in this case were consistent over the three different final states in question.

## Relative Strengths of the Compound Nucleus and Direct Interaction Modes of Reaction and Cross-Section Fluctuations

The theoretical angular distributions calculated with the DWBA can be used to obtain approximate values of the ratio of the DI and CN components of the reaction probability. Assuming the validity of the DWBA predictions, the minimum necessary amount of CN component indicated for any particular reaction would be the difference between the experimental cross section and the normalized theoretical cross section. With the

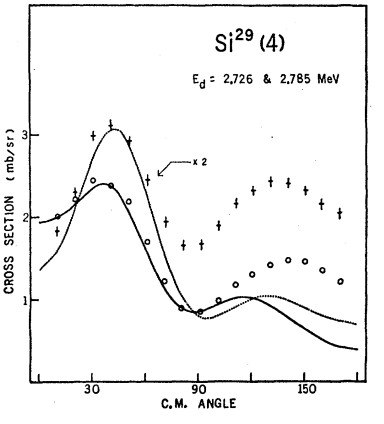


FIG. 23. The data points of the $Si^{28}(d,p)Si^{29}(4)$ reaction at $E_d=2.785$ MeV are represented by the cross marks. The open circles represent the data of the same reaction at $E_d=2.726$ MeV. The solid curve is computed for $Si^{29}(4)$ at $E_d=2.750$ MeV with parameter set C. The dotted line represents the same computation based upon the parameter set D.

requirement that the DI component vary smoothly with energy, this method yields a consistent method for the extraction of the DI–CN ratio, but one which probably predicts too little compound-nucleus formation. At cross-section minima, the CN components obtained average about 20% of the total cross section. At cross section maxima, this value varies from 60 to 70%. The $Si^{29}(10)$ reaction, characterized by a single particle final state, has lower predicted CN admixtures, ranging from 10 to 30%.

A similar approach to the problem is to subtract an isotropic term from the experimental angular distribution so that the relative heights of the peaks best approximate the theoretical predictions. The magnitude of the CN component is then obtained from the sum of the isotropic term and any remaining excess experimental intensity. This is a variation of the approach of Kuehner *et al.*[2] who used CN distributions computed from classical statistical theory (symmetric about 90°) and a DI component computed by the PWBA. They obtained approximate values for the CN contribution to the total cross section of 15% for $Si^{29}(0)$ and 30% for $Si^{29}(1)$ and $Si^{29}(2)$ in the deuteron energy range from 6 to 10 MeV. The assumptions of this approach are particularly weak at the lower energies of the present experiment because the principal stripping peaks are relatively smaller, and because isotropy or symmetry in the CN distribution[7,16] is the exception rather than the rule. Also, the predictions of the DWBA for the relative differential cross section at back angles, although much more realistic than the corresponding PWBA values, are sensitive to small changes in the parameters, and hence, to some extent unreliable. With these explicit reservations, however, this procedure may be employed to estimate possible corrections to the minimum values of the CN contribution obtained previously. The results for the $l_n=0$ transitions are not appreciably affected by this alternate approach, while the estimated proportion of the CN component of the $l_n=1$ and 2 transitions is increased by up to a factor of 2.

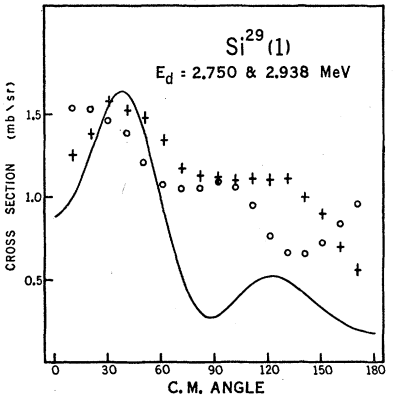


FIG. 24. The cross marks indicate the data from the $Si^{29}(1)$ reaction at $E_d=2.938$ MeV and the circles mark the data from the same reaction at $E_d=2.750$. The solid line represents the DWBA predictions for this reaction at $E_d=3.001$ MeV, based upon parameter set D.

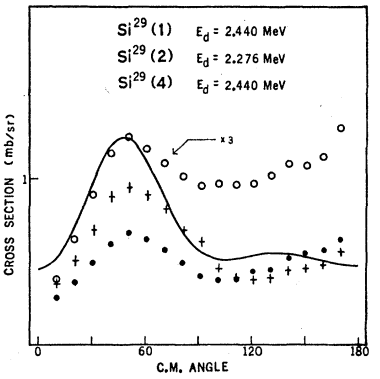


FIG. 25. $l_n=2$ transitions below $E_d=2.5$ MeV. The open circles indicate data of the $Si^{29}(4)$ reaction, the filled circles indicate data of the $Si^{29}(2)$ reaction, and the cross marks indicate data of the $Si^{29}(1)$ reaction. The solid line represents the DWBA prediction for the $Si^{29}(4)$ reaction at 2.440 MeV, based upon parameter set E.

The extent to which experimental observations of $(d,p)$ reactions on medium light nuclei can be interpreted in terms of Ericson fluctuations[16] of the CN or evaporative component of the reaction probability is of some interest.[35] The excitation energies in the compound states $P^{30}$ and $P^{32}$ which are reached in the present experiment are approximately 14–15 MeV in $P^{30}$ and 15–16 MeV in $P^{32}$. With the assumption[16] that the level densities of these systems follow the simple empirical formula,[16] $N(E)=e^{\alpha E}$, approximate values for the level densities in regions of interest were obtained from compilations of experimental data on energy levels of $P^{30}$ and $P^{32}$. A conservative prediction for the level densities at 14 MeV would appear to lie between 0.5 and 5.0 levels per keV. The fact that the probable upper limit of the angular momentum in the incident channel is $l=3$ would perhaps reduce the effective density somewhat. Therefore, the experimental energy resolution ($\sim$5 keV) probably encompasses several levels of the compound system.

Ericson fluctuations should occur when the level widths and spacings are such that, at any particular energy, several levels are excited even with infinite experimental resolution, e.g., $\Gamma$, the "width" or "coherence energy," is greater than $D$, the average level spacing. If the experimental energy resolution, $\Delta E_i$, is such that $\Delta E_i<\Gamma$, the cross section is predicted to vary qualitatively according to

$$\sigma_{\alpha\alpha'}=\langle\sigma_{\alpha\alpha'}\rangle_{\mathrm{av}}[1\pm 1/(nN)^{1/2}],$$

where $\langle\sigma_{\alpha\alpha'}\rangle_{\mathrm{av}}$ is the average cross section, $N$ the number of compound states excited, and $n$ the number of final states considered. This implies that the fluctuations

[35] L. Colli, in *Direct Interactions and Nuclear Reaction Mechanisms*, edited by E. Clementel and C. Villi (Gordon and Breach, New York, 1963), p. 397.

with energy of the cross sections of the various final states are uncorrelated so that an average over final states tends to smooth the data. Cross sections to individual final states fluctuate from state to state at a given incident energy. Angular distributions are asymmetric, tending toward symmetry when averaged over final states or over the incident energy such that $\Delta E \gg \Gamma$. The width of the fluctuations is a measure of the amount the incident energy must be varied in order to populate a new ensemble of compound levels. This factor is directly related to the average width of the levels and hence the energy widths of the cross-section fluctuations yield a rough measure of the lifetimes of the compound states participating in the reaction.

The characteristics of the data from the present experiment are quite similar to these predictions. The cross sections of the various reactions fluctuate independently. One of the many examples of this behavior is exhibited at $E_d = 2.010$ MeV where the $Si^{29}(1)$ reaction passes through an extreme maximum while the cross sections for the other four low-lying states are almost constant through the same region. Averaging the cross sections over the first five states smooths the fluctuations to a large extent, as would be expected.

It is possible that the observed fluctuations could result largely from interference between the CN and DI modes of reaction. This interference could produce effects similar to those predicted on the basis of interference between the overlapping CN levels alone.[16] It must be pointed out again that there is no *a priori* basis for supposing that interference between the different modes is negligible.[7] Two results from the present data indicate, however, that the CN component alone still fluctuates significantly and that possibly the CN–DI interference is not a major factor. The $Si^{29}(3)$ reaction does not involve any known DI mechanism according to all available evidence.[29,31] The angular distributions of this reaction are asymmetric and the cross sections fluctuate in a manner closely resembling the data from the other reactions to low-lying states of $Si^{29}$. This behavior apparently cannot have its origins in any DI–CN interference, but must originate within the CN mechanism itself, in the manner prescribed by Ericson.[16] The $Si^{29}(10)$ reaction, at the opposite extreme, is expected to have a predominant DI component due to the single-particle characteristics of the final state. The division of the total cross section into CN and DI components, based upon the DWBA analysis, indicates that the CN component constitutes 10–30% of the total cross section. This estimate is in fair agreement with the result $\mathrm{CN/DI} = \frac{1}{4}$ which follows from the assumption that the CN component is approximately as strong in the $Si^{29}(10)$ reaction as it is in the $Si^{29}(3)$ reaction. If, to be conservative, the CN reaction probability is assumed to be only one-tenth of the total reaction cross section, a very crude estimate of the fluctuations to be expected from possible CN–DI interference can be obtained by assuming that the total cross section is proportional to

$$|(\mathrm{CN})_{\mathrm{ampl.}}+(\mathrm{DI})_{\mathrm{ampl.}}|^2 \rightarrow |(\mathrm{CN})_{\mathrm{ampl.}}{}^2| + |(\mathrm{DI})_{\mathrm{ampl.}}|^2 \pm 2|(\mathrm{CN})_{\mathrm{ampl.}}||(\mathrm{DI})_{\mathrm{ampl.}}|.$$

The cross section should thus fluctuate between the limits of $\pm 60\%$ of the average cross section if the DI and CN amplitudes are in the assumed ratio of $(10)^{1/2}$ to 1. The largest observed deviations from straight line fits to the differential and total cross-section data of the $Si^{29}(10)$ reaction are approximately 15%.

Previous higher energy measurements[2] on $Si^{28}(d,p)Si^{29}$ employed energy resolution of approximately 120 keV and observed fluctuation widths as narrow as the experimental resolution. With resolution in the present experiment of 3–5 keV and with energy increments of approximately 20 keV, significant fluctuations occur having a half-widths from 40 to 150 keV. Trial measurements of excitation functions employing 5-keV steps indicated that no finer structure was overlooked.

The more quantitative predictions of fluctuation theory[36] can be employed to analyze the data if one assumes that the observed phenomena are basically of the Ericson type. One quantity of interest is the correlation function defined by

$$C(\epsilon) = \frac{\langle d\sigma(\theta,E)d\sigma(\theta,E+\epsilon)\rangle_{\mathrm{av}}}{\langle d\sigma(\theta,E)\rangle_{\mathrm{av}}{}^2} - 1,$$

where $d\sigma(\epsilon)$ is the differential cross section at some angle $\theta$ and the averages are over the incident energy. In its most simple form,[37] $C(\epsilon)$ as a function of the fundamental quantities of interest is expressed by

$$C(\epsilon) = \frac{1}{W}[1-y_{\mathrm{DI}}{}^2]\frac{\Gamma^2}{\Gamma^2+\epsilon^2},$$

where $y_{\mathrm{DI}}$ is the amount of DI component proportional to the total average cross section and $W$ is approximately equal to $\frac{1}{2}(2I_I+1)(2I_T+1)(2I_E+1)(2I_R+1)$, the spins referring to the incident, target, emitted, and residual particles. An average cross-section value that changes with energy complicates this analysis and, correspondingly, more attention was bestowed on those excitation functions which exhibited relatively constant average values.

The low value of $W$ for the $Si^{29}(0)$ reaction makes it of primary interest. The excitation function of $Si^{28}(d,p)Si^{29}(0)$ at 135° yields a value of $C(0) = 0.21$ and the data at 30° a value $C(0) = 0.20$. The value of $\Gamma$, obtained from the half-width of the first maximum of the plot of $C(\epsilon)$ versus $\epsilon$, is approximately $37 \pm 4$ keV. The data of the $Si^{29}(1)$ reaction at 135° yield $C(0) = 0.10$; that of $Si^{29}(2)$ at 30°, an instance of increasing average cross section, yields $C(0) = 0.146$. If one assumes $W$ as given

[36] T. Ericson, Ann. Phys. (N. Y.) **23**, 390 (1963).
[37] B. W. Allardyce, W. R. Graham, and I. Hall, Nucl. Phys. **52**, 239 (1964).

TABLE II. Legendre polynomials least-squares fits to experimental angular distributions.

| State | $E_d$ (MeV) | $a_1/a_0$ | $a_2/a_0$ | $a_3/a_0$ | $a_4/a_0$ | $a_5/a_0$ | $a_6/a_0$ | $a_7/a_0$ | $a_8/a_0$ | $a_9/a_0$ | $a_{10}/a_0$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $Si^{29}(0)$ | 3.002 | 0.146 | 0.006 | −0.719 | 0.948 | 0.518 | 0.689 | 0.092 | 0.095 | 0.038 | 0.008 |
| $Si^{29}(0)$ | 2.785 | 0.558 | 0.216 | −0.098 | 1.114 | 1.003 | 0.545 | 0.295 | 0.087 | 0.037 | 0.005 |
| $Si^{29}(0)$ | 2.750 | 0.714 | 0.141 | 0.462 | 1.400 | 0.901 | 0.529 | 0.180 | 0.074 | 0.032 | 0.022 |
| $Si^{29}(0)$ | 2.726 | 0.631 | 0.278 | 0.533 | 1.273 | 1.128 | 0.755 | 0.405 | 0.205 | 0.110 | 0.034 |
| $Si^{29}(1)$ | 2.938 | 0.269 | 0.009 | 0.088 | −0.216 | −0.065 | −0.080 | −0.005 | 0.011 | −0.010 | −0.017 |
| $Si^{29}(1)$ | 2.785 | 0.263 | 0.319 | −0.321 | −0.129 | −0.168 | −0.004 | −0.016 | −0.003 | −0.063 | 0.006 |
| $Si^{29}(1)$ | 2.750 | 0.126 | 0.238 | −0.020 | 0.082 | −0.143 | 0.006 | −0.004 | 0.006 | −0.012 | −0.001 |
| $Si^{29}(1)$ | 2.726 | 0.357 | 0.086 | 0.037 | 0.225 | −0.179 | −0.059 | 0.010 | 0.006 | 0.001 | 0.022 |
| $Si^{29}(10)$ | 3.002 | 0.240 | −0.118 | −0.128 | 0.058 | −0.053 | −0.159 | −0.155 | −0.100 | −0.039 | −0.012 |
| $Si^{29}(10)$ | 2.938 | 0.247 | 0.086 | −0.082 | 0.106 | −0.180 | −0.233 | −0.141 | −0.040 | −0.029 | −0.023 |

previously, the approximate values of $1/W$ for the $Si^{29}(0)$, $Si^{29}(1)$ and $Si^{29}(2)$ reactions are, respectively, 0.17, 0.08, and 0.06. These values are all smaller than the experimental values of $C(0)$, which should equal $(1/W)(1-y_{DI}^2)$. The $Si^{29}(3)$ reaction, with values of $C(0)=0.24$ at 30° (average cross section increasing) and $C(0)=0.12$ at 135° and $1/W=0.08$-confirms the implication that the effective value of $W$ must be less than is indicated by the spin-weight formula.

The distribution of cross-section values, plotted as a function of $\eta=\sigma/\sigma_{av}$, can yield information about the effective value of $W$. If one assumes that the DI component is small, $W=1/(1-\eta_m)$ where $\eta_m$ is the most probable value of $\eta$.[37] In order to compensate for changes in average cross section, deviations were measured about the least-square straight-line fit to the excitation functions. The $Si^{29}(0)$ reaction yields a value of $\eta_m=0.7$ for both the 135° and, less clearly, the 30° data. This result yields $W\approx 3$. Both excitation functions of the $Si^{29}(3)$ reaction also indicate $W\approx 3$. The $Si^{29}(1)$ and $Si^{29}(2)$ reactions considered do not yield an unambiguous value for $\eta_m$ and the cross sections are distributed roughly evenly about the mean. The available information is thus insufficient to clearly determine a consistent correction to $W$. We will assume that the effective value of $W$ is two-thirds the value suggested by the formula previously stated, a value intermediate to those suggested by the most reliable data from the $Si^{29}(0)$ and $Si^{29}(3)$ reactions.

With this assumption for the effective value of $W$, values for $y_{DI}$ can now be obtained. The predicted ratio of the DI component to the average CN component of the $Si^{29}(0)$ reaction at 30 and 135° is approximately 45–55. A similar ratio is obtained for the $Si^{29}(1)$ reaction at 135°. The $Si^{29}(2)$ data indicate no DI component, an aberration attributable to the high value of $C(0)$ that arose in part from the changing average cross section. The $Si^{29}(3)$ data at 135° indicate no DI component. The $Si^{29}(10)$ reaction was treated over a smaller energy range in an attempt to minimize the effects of its increasing cross section. It was found that $C(0)=0.015$, corresponding to a DI/CN ratio greater than 15 to 1. Obviously, smaller DI components in all cases would be predicted for larger effective values of $W$.

## Other Aspects of Low-Energy $(d,p)$ Reactions

### *Legendre Polynomial Expansions of the Experimental Angular Distributions*

Typical samples of the angular-distribution data were expanded in Legendre polynomials by a least-squares procedure using a computer program checked for accuracy through the $P_{10}$ term.[19] The motivation for this treatment of the data was to establish an upper limit on the angular momentum of the incident partial waves participating in the reaction. A significant nonzero coefficient of the polynomial of order $2L$ indicates that partial waves of order $l$ play a significant part in the reaction.[38] The expansion coefficients for typical angular distributions are presented in Table II. There is no significantly large coefficient for terms higher than order six or seven. Hence, the highest partial waves that are effective in this energy range appear to be of orbital angular momentum $l=3$. At higher bombarding energies ($E_d=6$–10 MeV), the higher order partial waves play a more prominent role in this reaction.[2] The limitations on the orbital angular momentum of the incident particles which enter the reaction in the present experiment would seem to indicate that the same partial waves might contribute significantly to both the DI and CN processes. The present situation might thus differ from that at higher energies, where apparently the DI reaction component is formed with the higher order incident waves and the CN component proceeds with those of lower order. The fact that, to some extent, the same partial waves enter into both reaction processes makes the postulation of incoherent addition of the CN and DI components less tenable. The hypothesis that the cross-section fluctuations result largely from CN–DI interference[2] could be extended to account for the total cross-section variations observed in the present experiment since the interference effects do not disappear in the total cross-section integrations if the same order waves occur in both reaction amplitudes. A definitive evaluation of the possible alternative explanations of the observed phenomena is thus very difficult.

[38] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons Inc., New York), p. 535.

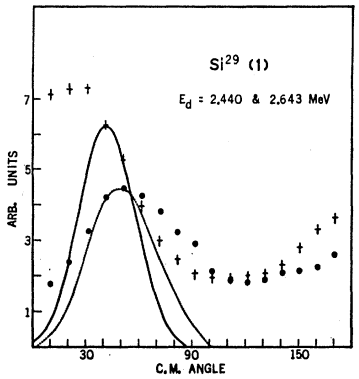


FIG. 26. The experimental points are from the $Si^{29}(1)$ reaction. The cross marks represent data obtained at $E_d=2.643$ and the circles represent data obtained at $E_d=2.440$ MeV. The solid line is a PWBA curve computed for $l_n=2$, $R=6.6$ F. The dotted line is the PWBA curve for $l_n=2$, $R=5.4$ F.

### *PWBA Analysis*

The efficacy of the plane-wave Born approximation, or Butler theory[8] as a spectroscopic tool at low bombarding energies, and particularly for low $Q$-value reactions, has been discussed to some extent.[10,11,39] The theoretical explantion of the sometimes surprisingly good results of the PWBA in regions where the various distortions should be severe is questionable in the light of DWBA calculations.[40,41] The distributions predicted by the PWBA theory were calculated for the reactions of this experiment, using the formulation of French.[42] These calculations provided a check on the reliability of a PWBA, as opposed to a DWBA, analysis of the data. Many of the present experimental distributions were characterized by backward peaking, and the alternate stripping mechanisms, such as heavy particle stripping[43] and exchange stripping,[44] have often been proposed as possible causes for these effects. Recent investigations[45] seem to indicate, however, that these reaction mechanisms are not significant for nuclei in the range of silicon and they were therefore not considered.

The $l_n=0$ transitions are, of course, quite distinctive once the stripping pattern has developed. All distributions of $l_n=0$ transitions in the deuteron energy range from 2 to 3 MeV show the 0° maximum. Distributions obtained below $E_d=2$ MeV show that even the 0° peak disappears at some lower deuteron energy values. The stripping cross section is presumably negligible with respect to the compound-nucleus reaction cross section in this region.

The distributions predicted for the $l_n=2$ transitions to $Si^{29}$ and $Si^{31}$ by the PWBA have their peak intensities at approximately 45°. This value is in good agreement with several of the experimental distributions (Fig. 26). The peaks of distributions of the same reactions at neighboring energies, however, occur at such angles that they cannot be matched by the $l_n=2$ PWBA predictions for any reasonable choice of the interaction radius (Fig. 26). Some of these distributions are, in fact, more closely approximated by assuming, incorrectly, that $l_n=1$.

The $Si^{28}(d,p)Si^{29}(10)$ and the $Si^{30}(d,p)Si^{31}(6)$ reactions are particularly interesting because of their low $Q$ values, 1.322 and 0.824 MeV, respectively. It has been proposed that reactions having small $Q$ values might best agree with PWBA predictions.[39] The transition to $Si^{29}(10)$ definitely proceeds with $l_n=1$.[29,31] The experimental distributions between $E_d=2.7$ and 3.0 MeV exhibit a peak cross section at an angle intermediate between the PWBA predictions for $l_n=1$ and $l_n=2$ (Fig. 27). The value of $l_n$ for the $Si^{31}(6)$ transition has not been previously assigned. The experimental distribution is similar to that of $Si^{29}(10)$, but the peak occurs closer to 40°. A PWBA analysis again does not give a definitive result, but judging on that basis alone, the assignment could well be $l_n=2$. Actually, as was seen, the DWBA analysis gives a rather unambiguous assignment of $l_n=1$.

It is evident from these results that a PWBA analysis of data from low-energy $(d,p)$ reactions with medium light nuclei does not yield consistently correct results. The agreement of the theory with low $Q$-value reactions is particularly poor, even though the final states involved seem to possess a relatively strong single-particle character.

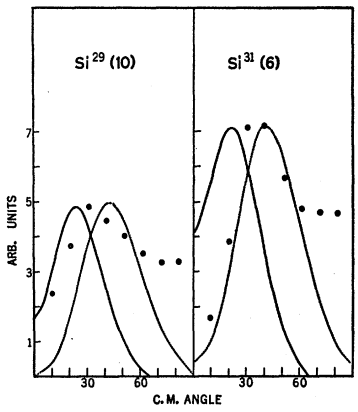


FIG. 27. $l_n=1$ transitions and PWBA curves. The $Si^{29}(10)$ data were obtained at $E_d=2.938$ MeV. The $Si^{31}(6)$ points are the average of the data from $E_d=2.938$ and 3.001 MeV. The solid lines are based upon $l_n=1$, $R=6.6$ F, the dotted lines upon $l_n=2$, $R=6.6$ F.

[39] D. H. Wilkenson, Phil. Mag. **3**, 1185 (1958).

[40] W. Tobocman, in *Proceedings of the International Conference on Nuclear Structure, Kingston, Ontario*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, 1960), p. 307.

[41] J. B. French, in *Proceedings of the Rutherford Jubilee International Conference Manchaster, 1961*, edited by J. B. Birks (Academic Press Inc., New York, 1961), p. 423.

[42] J. B. French, in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B, p. 890.

[43] G. E. Owen and L. Medansky, Phys. Rev. **105**, 1766 (1957).

[44] A. P. French, Phys. Rev. **107**, 1655 (1957).

[45] L. S. Rodberg, Nucl. Phys. **47**, 1 (1963).

## CONCLUSIONS

It appears that consistent quantitative agreement between the experimental angular distributions of $(d,p)$ reactions with medium light nuclei and the DWBA predictions for such reactions is unlikely. The difficulty seems to arise from the contribution to the experimental differential cross section from compound-nucleus formation. The angular distributions of this compound-nucleus component are fluctuating and asymetric and hence difficult to disentangle from the stripping distribution. At the low energies employed in this experiment, the direct interaction mechanism accounts for only approximately one-half of the observed cross section. The magnitudes of the CN effects are such that various anomalies observed in $(d,p)$ reactions at considerably higher energies can probably be explained as large CN contributions to the stripping distribution.[31,35]

With observations at a number of bombarding energies, individual experimental angular distributions may be obtained which apparently are relatively slightly affected by CN contamination. These experimental distributions can be fairly well approximated by DWBA predictions in a consistent manner. Multiple observations are especially necessary at low bombarding energies where the stripping distribution is not so sharply peaked. With these precautions, DWBA analysis of low-energy $(d,p)$ distributions would appear to yield valid spectroscopic data. Detailed confirmation of the proper values of the optical-model parameters remains, however, a difficult and uncertain problem because of the residual CN effects. An uncritical application of plane-wave Born-approximation analysis to $(d,p)$ reactions initiated with low-energy deuterons can yield misleading information concerning $l_n$.

There is some evidence that the rapid variations in cross sections and angular distributions observed in this experiment can be explained as Ericson-type fluctuations in the compound-nucleus component of the reactions. The role of interference between the compound nucleus and direct modes of interaction is difficult to assess, however, and may account for a considerable portion of the variation.

## ACKNOWLEDGMENTS

We wish to thank W. R. Smith for the use of his DWBA computer program. Gale I. Harris and David Breitenbecker provided invaluable assistance with the computations. We are grateful to the staff of the University of Kansas, Nuclear Physics Laboratory for aid in the experimental work.

---

PHYSICAL REVIEW VOLUME 135, NUMBER 3B 10 AUGUST 1964

# Approximation for the Phase Shifts Produced by Repulsive Potentials Strongly Singular in the Origin

FRANCESCO CALOGERO
*Istituto di Fisica dell'Università, Roma, Italy*
*and*
*Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Italy*
(Received 25 March 1964)

An approximate expression for the partial-wave scattering phase shifts produced by a potential strongly divergent at the origin is introduced. This approximation is applicable over the whole energy range; it yields the correct threshold behavior of the phase shifts and becomes exact at high energy. The high-energy behavior of the phase shifts $\delta_l$ is ascertained; we find, as $k$ diverges ($l$ fixed), $\delta_l = -Ak^{1-2/m}$, where $m$ characterizes the behavior of the potential at the origin, $V(r) \sim r^{-m}$, $m>2$, and $A$ is a constant independent of $l$ which is evaluated explicitly. Numerical tests of the accuracy of the approximation are given, including comparisons with the WKB approximation.

## I. INTRODUCTION

IN this paper we introduce and discuss an approximate expression for the scattering phase shifts produced by a repulsive potential strongly divergent in the origin. We concentrate our attention on $S$-wave scattering; the generalization to all partial waves is given at the end of the paper.

Singular potentials are important in phenomenological treatments of nuclear and atomic interactions. A second motivation for discussing scattering on singular potentials lies in the connection of this problem with that of the divergences in (renormalizable or unrenormalizable) field theories.

It is well known that for strongly divergent potentials scattering theory makes sense only if the divergent core is repulsive, because only in this case may the normalization of the wave function in the origin be maintained. This implies that the scattering amplitude will not depend in an analytic way upon the strength of the interaction. Thus, the Born approximation, being the first term in a power expansion in the strength of the potential, fails to converge; nor is Fredholm theory